Nama : Mohammad Febryan Khamim
NRP : 5002221072

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PDB Kelas C

Perbaikan Kuis 1

1. $(3x-2y)\frac{dy}{dx} = 3y$

$\Rightarrow 3y\,dx = (3x-2y)\,dy$

* Uji apakah termasuk PD Homogin

- $M(x,y) = 3y$

$M(\lambda x, \lambda y) = 3\lambda y = \lambda \cdot 3y$

- $N(x,y) = 3x-2y$

$N(\lambda x, \lambda y) = 3\lambda x - 2\lambda y = \lambda(3x-2y)$

$\Rightarrow 3y\,dx = (3x-2y)\,dy \quad \times \frac{1}{x}$

$3\frac{y}{x}\,dx = \left(3 - 2\frac{y}{x}\right)dy$

misal : $y = \vartheta x \rightarrow dy = d\vartheta \cdot x + \vartheta\,dx \quad : dx$

$\frac{dy}{dx} = \frac{d\vartheta}{dx}x + \vartheta \quad \ldots (1)$

Sehingga :

$3\vartheta\,dx = (3-2\vartheta)\,dy \quad \times \frac{1}{dx}$

$3\vartheta = (3-2\vartheta)\frac{dy}{dx}$

Substitusikan (1) :

$\frac{dy}{dx} = \frac{3\vartheta}{3-2\vartheta}$

$\frac{d\vartheta}{dx}x + \vartheta = \frac{3\vartheta}{3-2\vartheta}$

$\frac{d\vartheta}{dx}x = \frac{3\vartheta}{3-2\vartheta} - \frac{\vartheta(3-2\vartheta)}{3-2\vartheta}$

$= \frac{3\vartheta - (3\vartheta - 2\vartheta^2)}{3-2\vartheta}$

$= \frac{2\vartheta^2}{3-2\vartheta}$

$\frac{3-2\vartheta}{2\vartheta^2}\,d\vartheta = \frac{1}{x}\,dx$

* Integralkan kedua ruas :

$\int \frac{3-2\vartheta}{2\vartheta^2}\,d\vartheta = \int \frac{1}{x}\,dx$

* $\int \frac{3-2\vartheta}{2\vartheta^2}\,d\vartheta = \int \frac{3}{2\vartheta^2}\,d\vartheta - \int \frac{1}{\vartheta}\,d\vartheta$

$= \frac{3}{2}\cdot\left(-\frac{1}{1}\cdot\vartheta^{-1}\right) - \ln\vartheta$

$= -\frac{3}{2\vartheta} - \ln\vartheta$

Diperoleh :

$-\frac{3}{2\vartheta} - \ln\vartheta = \ln x + C$

$C = \ln x + \ln\vartheta + \frac{3}{2\vartheta}$

Substitusi $\vartheta = \frac{y}{x}$:

$C = \ln x + \ln\frac{y}{x} + \frac{3}{2\,y/x}$

$= \ln y + \frac{3x}{2y}$

Jawab : PUPD : $C = \ln y + \frac{3x}{2y}$

2). $\sin\left(\frac{y}{x}\right)\left[x\frac{dy}{dx} - y\right] = x\cos\left(\frac{y}{x}\right)$

$\left[x\sin\left(\frac{y}{x}\right)\right]\frac{dy}{dx} - y\sin\left(\frac{y}{x}\right) = x\cos\left(\frac{y}{x}\right)$

$\left[x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}\right] = x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right) \quad \times \frac{1}{x\sin\left(\frac{y}{x}\right)}$

$\frac{dy}{dx} = \cot\left(\frac{y}{x}\right) + \frac{y}{x}$

misal : $\vartheta = \frac{y}{x} \rightarrow y = \vartheta x$

$dy = d\vartheta \cdot x + dx \cdot \vartheta \quad \times \frac{1}{dx}$

$\frac{dy}{dx} = x \cdot \frac{d\vartheta}{dx} + \vartheta$

$\frac{dy}{dx} = \cot\left(\frac{y}{x}\right) + \frac{y}{x}$

$x\frac{d\vartheta}{dx} + \vartheta = \cot\vartheta + \vartheta$

$x\frac{d\vartheta}{dx} = \cot\vartheta$

$\frac{1}{\cot\vartheta}\,d\vartheta = \frac{1}{x}\,dx$

$\int \tan\vartheta\,d\vartheta = \int \frac{1}{x}\,dx$

* $\int \tan\vartheta\,d\vartheta = \int \frac{\sin\vartheta}{\cos\vartheta}\,d\vartheta$

misal : $\cos\vartheta = u$

$du = -\sin\vartheta\,d\vartheta$

$\int \frac{\sin\vartheta}{u}\cdot\frac{du}{-\sin\vartheta} = -\int \frac{1}{u}\,du$

$= -\ln u$

$\int \tan\vartheta\,d\vartheta = \int \frac{1}{x}\,dx$

$-\ln\cos\vartheta = \ln x + C$

$C = \ln x + \ln\cos\frac{y}{x}$

$C = \ln x\cos\left(\frac{y}{x}\right)$

$e^C = e^{\ln x\cos\left(\frac{y}{x}\right)}$

$C = x\cos\left(\frac{y}{x}\right)$

Jawab :

PUPD : $C = x\cos\left(\frac{y}{x}\right)$

3. $\frac{dy}{dx} + \frac{2}{x} y = 6y^2 x^4$

→ Merupakan PD Bernoulli

$\frac{dy}{dx} + \frac{2}{x} y = y^2 \cdot 6x^4$

misal : $z = y^{1-2}$ $\quad z' = -y^{-2}(y')$

$= y^{-1}$ $\quad \frac{dy}{dx} = -z' y^2$

⟹ $-z'y^2 + \frac{2}{x} y = y^2 6x^4 \quad \times -y^{-2}$

$\frac{dz}{dx} - \frac{2}{x} \cdot \frac{1}{y} = -6x^4$

$\frac{dz}{dx} + \left(-\frac{2}{x}\right) z = -6x^4$

$F(x) = e^{\int P(x) dx}$

$= e^{\int -\frac{2}{x} dx}$

$= e^{-2\ln x}$

$= e^{\ln x^{-2}}$

$= \frac{1}{x^2}$

PD Linier tingkat 1 :

$z \frac{1}{x^2} = \int -6x^4 \cdot \frac{1}{x^2} dx$

$\frac{z}{x^2} = -\frac{6}{3} x^3 + C$

$z = -2x^5 + Cx^2$

$\frac{1}{y} = -2x^5 + Cx^2$

$2x^5 + \frac{1}{y} = Cx^2 \quad \times \frac{1}{x^2}$

$2x^3 + \frac{1}{x^2 y} = C$

Jawab : PUPD $\equiv C = 2x^3 + \frac{1}{x^2 y}$

4. $\frac{xy-1}{x} dx + \frac{xy+1}{y} dy = 0$

$\left(y - \frac{1}{x}\right) dx + \left(x + \frac{1}{y}\right) dy = 0$

※ Buktikan PD Eksak

① $\frac{\partial M(x,y)}{\partial y} = 1 \qquad \frac{\partial N(x,y)}{\partial x} = 1$ (terbukti)

② $\frac{\partial F}{\partial x} = M(x,y)$

$F(x,y) = \int^x M(x,y) dx + R(y)$

$= \int^x y - \frac{1}{x} dx + R(y)$

$= xy - \ln x + R(y)$ ... ①

③ $\frac{\partial F}{\partial y} = x - 0 + R'(y)$

$x + \frac{1}{y} = x + R'(y)$

* $R'(y) = \frac{1}{y}$

$R(y) = \int \frac{1}{y} dy$

$= \ln y + C$

Sehingga :

$F(x,y) \equiv xy - \ln x + \ln y = C$

$C = xy + \ln \frac{y}{x}$

Jawab : PUPD : $C = xy + \ln \frac{y}{x}$

5). Diketahui : $t_0 = 4$ sore $\quad T_m = 75°F$

$T(10) = 415°F \quad ; \quad T(20) = 347°F$

Ditanya : a. $T(0)$

b. Jam berapa saat $T = 100°F$

Jawab : $\frac{dT}{dt} = -k(T - T_m)$

$\frac{dT}{dt} = -k(T - 75)$

$\int \frac{1}{T-75} dT = \int -k\, dt$

$\ln(T-75) = -kt + C$

$e^{\ln(T-75)} = e^{-kt+C}$

$T - 75 = e^C \cdot e^{-kt}$

$T = 75 + Ce^{-kt}$

• $T(10) = 75 + Ce^{-10k} = 415$

$Ce^{-10k} = 340$ ... ①

• $T(20) = 75 + Ce^{-20k} = 347$

$Ce^{-20k} = 272$ ... ②

Dari ① dan ② diperoleh :

$Ce^{-10k} = 340$	$\times e^{-10k}$	$Ce^{-20k} = 340e^{-10k}$
$Ce^{-20k} = 272$	$\times 1$	$Ce^{-20k} = 272$

$340e^{-10k} = 272$

$e^{10k} = \frac{340}{272}$

$\ln e^{10k} = \ln 1{,}25$

$10k = 0{,}223144$

$k = 0{,}0223144$

※Substitusi $k = 0,0223144$ ke (1) untuk dapatkan nilai C

$$Ce^{-10k} = 340$$
$$Ce^{-0,223144} = 340$$
$$C = 340\,e^{0,223144}$$
$$= 340 \cdot 1,25$$
$$= 425$$

a). $T(0) = 75 + Ce^{0}$
$$= 75 + 425$$
$$= 500^{\circ}F$$

b). $T(t) = 75 + Ce^{-kt} = 100$
$$425e^{-0,0223144t} = 25$$
$$e^{0,0223144t} = 17$$
$$\ln e^{0,0223144t} = \ln 17$$
$$0,0223144t = 2,8332133$$
$$t = \frac{2,8332133}{0,0223144}$$
$$= 126,96 \approx 127$$

127 menit setelah jam 4 sore

$\Rightarrow$ 18.07

Jadi, mencapai suhu $100^{\circ}F$ saat pukul 18.07

6) Isoterm, setengah bidang atas $(y>0)$ oleh persamaan $4x^2 + 9y^2 = C$. Temukan lintasan ortogonal

$$\Rightarrow 8x\,dx + 18y\,dy = 0$$
$$18y\,dy = -8x\,dx$$
$$\frac{dy}{dx} = \frac{-4x}{9y}$$

Sehingga PD Trayektori ortogonalnya :

$$\frac{dy}{dx} = \frac{-1}{-4x/9y}$$
$$\frac{dy}{dx} = \frac{9y}{4x}$$
$$\int \frac{1}{9y}\,dy = \int \frac{1}{4x}\,dx$$
$$\frac{1}{9}\ln y = \frac{1}{4}\ln x + C$$
$$C = \ln y^{1/9} - \ln x^{1/4}$$

$$e^{C} = e^{\ln \frac{y^{1/9}}{x^{1/4}}}$$
$$C = \frac{y^{1/9}}{x^{1/4}}$$
$$y^{1/9} = Cx^{1/4}$$
$$y = Cx^{9/4}$$
$$y = Cx^{2} \cdot \sqrt[4]{x}$$